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Supersymmetry and Brane Cosmology

Antonio Riotto [a,b,1] and Luca Scarabello [a,2]

[a] *University of Padova, Department of Physics*

Via Marzolo 8, I-35131 Padova, Italy

[b] *INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padova, Italy*

Abstract

We consider a five-dimensional brane world scenario where the fifth dimension is compactified on S^1/Z_2. We show that the familiar four-dimensional cosmology on our brane is easily recovered during a primordial stage of inflation if supersymmetry is exploited. Even if some vacuum energy density appears localized on our three brane, heavy supersymmetric bulk fields adjust themselves and acquire a nontrivial configuration along the extra-dimension. This phenomenon redistributes uniformly the energy density across the bulk and the resulting energy-momentum tensor does not display any singularity associated to the initial localized energy density on our three-brane. No jumps across the brane are present for the derivatives of the metric and Einstein's equations are solved by constant solutions along the fifth dimension. Our findings make it clear that cosmological phenomena in the supersymmetric brane world scenario must be studied taking properly into account bulk supersymmetric states. This comment is particularly relevant when applied to (super)gravity since in supersymmetric brane world scenarios, even though chiral matter and gauge fields may be restricted to live on boundaries, gravity multiplets always propagate in the bulk.

[1]E-mail: antonio.riotto@pd.infn.it

[2]E-mail: luca.scarabello@pd.infn.it

1 Introduction

The goal of this paper is to investigate the impact of supersymmetry on the cosmology of the brane world. The recent exciting developments in string theory and the idea that our Universe may be thought as a three-brane embedded in a higher dimensional theory has lately stimulated a lot of activity in various fields of research. In the cosmological setting, it has been shown that a non-standard cosmological evolution of our Universe is induced if matter with energy density ρ is confined on three branes [1, 2]. The Friedmann equation governing the rate of the expansion of our three-brane Universe H_0 is modified and one finds $H_0^2 \propto \rho^2$, instead of the conventional four-dimensional Hubble law $H_0^2 \propto \rho$. This result is essentially due to nontrivial constraints on the derivatives of the scale factor along the extra-dimensions and on the energy densities when the latter are localized on the branes. A strong constraint on the brane world idea would then be provided by the requirement of having a standard cosmological evolution which successfully describes our Universe from the epoch of nucleosynthesis to the present day.

An elegant solution to this problem is offered within the Randall-Sundrum setting [3, 4] where the tension of the brane is compensated by a negative cosmological constant in the bulk. The standard Hubble law is almost recovered if the energy density on the brane is much smaller than the brane tension [5]. Indeed, in the phenomenologically interesting model which solves the hierarchy problem (in which our Universe is identified with a three-brane with negative tension), there is a crucial sign difference in the Friedmann equation. This obstacle is futher overcome if one takes into account that the so-called radion, the four-dimensional modulus parametrizing the radius of compactification, has to be stabilized [6, 7]. This clarifies that the origin of the unconventional cosmology is not due to the breakdown of the effective four-dimensional theory, but rather to a constraint that matter on the branes is forced to obey in order to ensure a static radion modulus. Upon radion stabilization, solutions can be found for in 5D for the 3-space scale factor $a(t, y)$ which have a nontrivial dependence on the coordinate y of the extra-dimension and a local minimum at some point $y = y_m$. If the theory is compactified on a circle with radius smaller than $|y_m|$, normal Friedmann expansion is obtained.

In this paper we will show that the familiar four-dimensional Hubble law in the brane world scenario can be recovered – at least during a primordial stage of inflation – if one exploits supersymmetry. This result has a simple explanation. Suppose, for instance, that one starts from a gauge theory extended to the entire bulk. If supersymmetry is imposed, the theory necessarily comes with heavy scalar fields which are contained in the supersymmetric multiplets and do not have a massless zero mode. They are coupled to the fields on the three-brane only through derivatives along the extra-dimension coordinates.

If some energy density appears localized on our three brane, these fields adjust themselves and acquire a nontrivial configuration along the extra-dimensions. This back-reaction redistributes uniformly the energy density across the bulk. In the effective four-dimensional theory the energy-momentum tensor does not display any singularity which would signal a localized source on our three-brane. Einstein's equations admit constant solutions for the scale factor across the bulk and the familiar four-dimensional Hubble law may be recovered.

Admittedly, this gratifying result has been found only for the specific epoch of the evolution of the Universe when the latter undergoes a period of accelerated expansion. Nevertheless, an important lesson can be learned from our simple exercise. When investigating various phenomena occured in the early Universe within the brane world scenario – inflation, reheating after inflation, phase transitions, generation of the baryon asymmetry, etc. – a careful treatment is needed to properly take into account all the degrees of freedom of the theory. Even those states which are massive and might seem irrelevant in the 4D effective description may play a crucial role and significantly alter (and possibly simplifying) the description of the cosmological evolution. This remark holds especially for those states living in the (super)gravity multiplets which are necessarily present in any supersymmetric construction of the brane world. We will come back to this point in the last section and discuss explicit examples.

The paper is organized as follows. In section 2 we consider a simple supersymmetric model of inflation whose dynamics is entirely confined on a boundary wall and briefly summarize the findings of Refs. [6, 7] to show how the standard 4D cosmological evolution may be obtained. In section 3 we present our results staring from a theory where supersymmetry is extended to the whole bulk. Section 4 contains our conclusions and a discussion of the implications of our findings.

2 Conventional cosmology in the brane world

Our starting point is a five-dimensional theory compactified on an orbifold S^1/Z_2 of (comoving) radius R. One writes the Lagrangian as

$$S = \frac{1}{2}\int d^4x \int_{-\pi R}^{+\pi R} dy\,\sqrt{g_5}\left\{\mathcal{L}_{\rm bulk} + \sum_i \delta(y-y_i)\mathcal{L}_{4i}\right\}\,. \tag{1}$$

The sum includes the walls at the orbifold points $y_i = 0, \pi R$. The bulk Lagrangian $\mathcal{L}_{\rm bulk}$ includes the standard 5D gravity Lagrangian

$$\mathcal{L}_{\rm bulk} = -\frac{1}{2}M_5^3 R_5\,, \tag{2}$$

where M_5 is the five-dimensional reduced Planck mass and R_5 is the five-dimensional scalar curvature. The five-dimensional metric is written as by

$$ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu - b^2(t)dy^2 = n^2(y,t)dt^2 - a^2(y,t)d\vec{x}^2 - b^2(t)dy^2\,, \tag{3}$$

where the five-dimensional coordinates are indicated by $x^M = (x^\mu, y)$ and $g_{\mu\nu}$ denotes the usual four-dimensional metric on hypersurfaces of fixed y. The latter parametrizes the extra dimension compactified on the interval $[-\pi R, +\pi R]$ and the Z_2 symmetry $y \leftrightarrow -y$ is imposed. Our four-dimensional brane world is supposed to be at $y = 0$.

Under the aforementioned decomposition (3) and after a conformal transformation $g_{\mu\nu} \to b^{-1} g_{\mu\nu}$, the action (1) can be written as

$$S = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_4} \left\{ -\frac{1}{2} M_5^3 \left[R_4(t,y) - \frac{3}{2} \dot{r}^2 \right] + e^{-r} \mathcal{L}_{\rm st} \right\} , \tag{4}$$

where R_4 is the four dimensional scalar curvature and we have explicitly inserted a Lagrangian $\mathcal{L}_{\rm st}$ which is resposible for the stabilization of the radion field $r \equiv \ln b$.

Einstein's equations are given by (after radion stabilization)

$$\begin{aligned}
G_{00} &= 3\left(\frac{\dot{a}}{a}\right)^2 - 3\frac{n^2}{b^2}\left[\frac{a''}{a} + \left(\frac{a'}{a}\right)^2\right] = \frac{T_{00}}{M_5^3} , && (5)\\
G_{ii} &= \frac{a^2}{n^2}\left[-\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\dot{a}}{a}\frac{\dot{n}}{n} - 2\frac{\ddot{a}}{a}\right] + \frac{a^2}{b^2}\left[\left(\frac{a'}{a}\right)^2 + 2\frac{a'}{a}\frac{n'}{n} + 2\frac{a''}{a} + \frac{n''}{n}\right] = \frac{T_{ii}}{M_5^3} , && (6)\\
G_{55} &= 3\left[\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right) + \frac{\ddot{a}}{a}\right)\right] = \frac{T_{55}}{M_5^3} , && (7)\\
G_{05} &= 3\left[\frac{n'}{n}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a}\right] = \frac{T_{05}}{M_5^3} , && (8)
\end{aligned}$$

where dot denotes differentiation whith respect to t, prime with respect to y and T_{MN} is the energy-momentum tensor.

2.1 Inflation driven by a boundary vacuum energy

Since we are interested in the case in which our Universe goes through an inflationary stage, we first assume that there is a nonvanishing vacuum energy V on our brane at $y = 0$. The corresponding energy-momentum tensor can be expressed in the form

$$T^A_{\ B}\Big|_{\rm brane} = \frac{\delta(y)}{b} \, {\rm diag}\,(V, V, V, V, 0) \ . \tag{9}$$

To be concrete, we suppose that inflation is driven by a nonvanishing supersymmetric D-term [8] (the same considerations hold for F-term inflation). To exemplify the description, let us consider an abelian $U(1)$ gauge theory on our brane (therefore gauge fields do not propagate in the bulk) with coupling constant g. The theory contains three chiral superfields on the boundary at $y = 0$: S, Φ_+ and Φ_- with charges equal to 0, +1 and -1 respectively under the $U(1)$ gauge symmetry. The superpotential on the boundary has the form

$$W = \lambda S \Phi_+ \Phi_- \tag{10}$$

and the Lagrangian contains the Fayet-Iliopoulos D-term

$$\mathcal{L}_{\rm FI} = D\,\xi \, . \tag{11}$$

The scalar potential in the global supersymmetry limit reads

$$V = \lambda^2|S|^2\left(|\phi_-|^2 + |\phi_+|^2\right) + \lambda^2|\phi_+\phi_-|^2 + \frac{g^2}{2}\left(|\phi_+|^2 - |\phi_-|^2 + \xi\right)^2 \tag{12}$$

where $\phi_\pm$ are the scalar fields of the supermultplets $\Phi_\pm$.

The global minimum is supersymmetry conserving, but the gauge group $U(1)$ is spontaneously broken

$$\langle S\rangle = \langle\phi_+\rangle = 0, \quad \langle\phi_-\rangle = \sqrt{\xi}\,. \tag{13}$$

However, if we minimize the potential, for fixed values of S, with respect to other fields, we find that for $S > S_c = \frac{g}{\lambda}\sqrt{\xi}$, the minimum is at $\phi_+ = \phi_- = 0$. Thus, for $S > S_c$ and $\phi_+ = \phi_- = 0$ the tree level potential has a vanishing curvature in the S direction and large positive curvature in the remaining two directions $m_\pm^2 = \lambda^2|S|^2 \pm g^2\xi$. For arbitrarily large S the vacuum energy density driving inflation is provided by the tree level value of the potential

$$V = \frac{g^2}{2}\xi^2 \tag{14}$$

and the S plays the role of the inflaton. Notice that under these circumstances the D-term $D = \xi + |\phi_+|^2 - |\phi_-|^2$ reduces to

$$D = \xi\,. \tag{15}$$

One-loop corrections generate an almost flat potential for the inflaton field S and the end of inflation is determined either by the failure of the slow-roll conditions or when S approaches S_c [8].

Since there is no flow of matter along the fifth dimension both T_{05} and G_{05} vanish. The $(0,5)$-component of Einstein's equations can be easily integrated to give

$$n(t,y) = \lambda(t)\,\dot{a}(t,y)\,. \tag{16}$$

The $(0,0)$-component of Einstein's equations reduces to a second-order differential equation for $a(t,y)$ while the function $\lambda(t)$ leads not only to the determination of the lapse function $n(t,y)$, but also to the four-dimensional Friedmann equation on our brane where the Hubble parameter can be expressed in terms of $\lambda(t)$ as

$$H_0^2 \equiv \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{1}{\lambda^2(t)a_0^2(t)}\,. \tag{17}$$

The brane can be taken into account by using the junction conditions which relate the jumps of the derivative of the metric across the brane to the stress-energy tensor (9) inside the brane [1, 2]. This gives

$$\frac{[a']}{a_0 b_0} = -\frac{1}{3}\frac{V}{M_5^3}\,, \tag{18}$$

where the subscript 0 for a_0 and b_0 means that these functions are taken in $y = 0$, and $[f] = f(0^+) - f(0^-)$ denotes the jump of the function f across $y = 0$.

The general solution for $a(t,y)$ can be written as

$$a^2(t,y) = a_0^2(t) + \alpha(t)\,|y| + \frac{b_0^2}{\lambda^2}\,y^2\,, \tag{19}$$

with

$$\alpha(t) = -\frac{a_0^2\,b_0}{3}\frac{V}{M_5^3}\,. \tag{20}$$

Notice that $a^2(t,y)$ has a minimum at $|y_m|(t) = -\alpha(t)\lambda^2(t)/2b_0^2$, which forces to compactify the extra dimension on a circle of radius $|y_m|$ by identifying the two extrema at $-|y_m|$ and $|y_m|$.

From Eq. (17) one derives [7]

$$H_0^2 = \frac{1}{\lambda^2(t)a_0^2(t)} = \frac{V}{6\,b\,|y_m|\,M_5^3} \tag{21}$$

from which it is concluded that one can recover the conventional 4D Friedmann equation only if $|y_m(t)| = $ constant. In such a case, one can identify the four-dimensional reduced Planck mass

$$M_4^2 = 2\,b\,|y_m|\,M_5^3\,. \tag{22}$$

Let us suppose that a bulk potential for the radion field $V(b)$ is generated in the five-dimensional theory by some mechanism and the radion is very heavy, that is if near the minimum b_0 we have $V(b) \simeq M_5^5\,(b-b_0)^2/b_0^2$ with a very high mass scale M_5. Since $\left(T^\mu_{\ \mu} - 2\,T^5_{\ 5}\right)$ is the source for the radion modulus, the latter remains in equilibrium if the energy momentum tensor satisfies the following constraint [7]

$$\int_{-|y_m|}^{+|y_m|} dy\,\sqrt{-g_4}\,e^{-r}\,\left(T^\mu_{\ \mu} - 2\,T^5_{\ 5}\right) = 0\,. \tag{23}$$

Since during the inflationary stage on our brane there is an extra vacuum energy density (14), the equilibrium position for the radion field changes. Using the constraint (23) (with the integration over the fifth coordinate y now going from $-|y_m|$ to $|y_m|$) and $T^\mu_{\ \mu}\Big|_{\rm brane} = 4\,\frac{\delta(y)}{b}\,V$, one finds that the minimum of the radion field is shifted by a small amount if V is much smaller than M_5^4 and that the (55)-component of the energy momentum tensor becomes (up to order $\mathcal{O}(V^2)$)

$$T^5_{\ 5} = \frac{V}{b_0\,|y_m|}\,. \tag{24}$$

Under these circummstances $y_m(t)$=constant, as one can easily check plugging the solution (19) into the (55)-component of Einstein's equations [7], and the Friedmann equation for our Universe becomes

$$H_0^2 = \frac{1}{3}\frac{V}{M_4^2}\,. \tag{25}$$

The conventional four-dimensional Hubble rate is recovered on our three-brane at the expense of limiting the space available in the extra-dimension and compactifying on a circle of radius $|y_m| \leq \pi R$.

Our goal is now to show that the conventional four-dimensional cosmology (at least during the inflationary stage) is recovered when making use of all the tools offered by supersymmetry. As we will show, our path towards standard 4D cosmology differs considerably from the one outlined in this section.

3 Supersymmetry and conventional four-dimensional cosmology

We consider a simple variant of the model of inflation discussed in the previous section and suppose that the abelian gauge theory $U(1)$ lives in the bulk. Gauge fields are therefore free to propagate in the extra dimension.

The five-dimensional $U(1)$ gauge multiplet with coupling constant g contains a vector field A^M, a real scalar field Φ, and a gaugino λ^i. The five-dimensional Yang-Mills multiplet is then extended to an off-shell multiplet by adding an $SU(2)$ triplet X^a of real-valued auxiliary fields. Here capitalized indices M, N run over 0,1,2,3,5, lower-case indices μ run over 0,1,2,3, and i, a are internal $SU(2)$ spinor and vector indices, with $i = 1, 2$, $a = 1, 2, 3$.

Now we have to project this structure down to a four dimensional $N = 1$ supersymmetry transformation acting on fields on the orbifold points. A generic bulk field $f(x^\mu, y)$ transforms under the action of the Z_2-symmetry as $f(x^\mu, y) = P\, f(x^\mu, -y)$ where P is an intrinsic parity equal to ± 1. The quantum number P must be assigned to fields in such a way that it leaves the bulk Lagrangian invariant. Then fields of $P = -1$ vanish on the walls but have nonvanishing derivatives $\partial_5 f$.

We assign even Z_2–parity to the fields

$$A^\mu,\ \lambda^1_L,\ X^3\,, \tag{26}$$

and odd Z_2–parity to the fields

$$A^5,\ \Phi,\ \lambda^2_L,\ X^1,\ X^2\ . \tag{27}$$

On the wall at $y = 0$, the five-dimensional supersymmetry transformations reduce to the following transformation of the even-parity states generated by ξ^1_L:

$$\begin{aligned}
\delta_\xi A^\mu &= i\xi^{1\dagger}_L \overline{\sigma}^\mu \lambda^1_L - i\lambda^{1\dagger}_L \overline{\sigma}^\mu \xi^1_L\,, \\
\delta_\xi \lambda^1_L &= \sigma^{\mu\nu} F_{\mu\nu} \xi^1_L - i(X^3 - \sqrt{-g^{55}}\partial_5\Phi)\xi^1_L\,, \\
\delta_\xi X^3 &= \xi^{1\dagger}_L \overline{\sigma}^\mu D_\mu \lambda^1_L + \xi^{1\dagger}_L \sigma^2 \sqrt{-g^{55}}\partial_5 \lambda^{2*}_L + \text{h.c.}\,, \\
\delta_\xi \sqrt{-g^{55}}\partial_5\Phi &= \xi^{1T}_L \sigma^2 \sqrt{-g^{55}}\partial_5 \lambda^2_L + \xi^{1\dagger}_L \sigma^2 \sqrt{-g^{55}}\partial_5 \lambda^{2*}_L\,.
\end{aligned} \tag{28}$$

The last two equations imply

$$\delta_\xi (X^3 - \sqrt{-g^{55}}\partial_5\Phi) = \xi^{1\dagger}_L \overline{\sigma}^\mu D_\mu \lambda^1_L + \text{h.c.}\,. \tag{29}$$

A simple inspection of these transformations reveals that the even fields A^μ, λ^1_L, and $(X^3 - \sqrt{-g^{55}}\partial_5\Phi)$ transform as the vector, gaugino, and the auxiliary D-field of a 4D $N=1$ vector multiplet [9].

It is then obvious how to couple the five-dimensional gauge multiplet to a generic 4D dimensional chiral multiplet living on the boundary and charged under the $U(1)$ symmetry [9]. One writes the Lagrangian as

$$S = \frac{1}{2}\int d^4x \int_{-\pi R}^{+\pi R} dy\, \sqrt{g_5}\left\{\mathcal{L}_{\rm gauge} + \sum_i \delta(y-y_i)\mathcal{L}_{4i}\right\}\ , \tag{30}$$

where the sum includes the walls at $y_i = 0, \pi R$. The bulk Lagrangian $\mathcal{L}_{\rm gauge}$ is the standard one for a 5D super-Yang-Mills multiplet

$$\begin{aligned}\mathcal{L}_{\rm gauge} &= \frac{1}{g^2}\left(-\frac{1}{2}{\rm Tr}\,(F_{MN})^2 + {\rm Tr}\,(D_M\Phi)^2 + {\rm Tr}\,(\overline{\lambda} i\gamma^M D_M\lambda)\right.\\ &+ \left.{\rm Tr}\,(X^a)^2 - {\rm Tr}\,(\overline{\lambda}[\Phi,\lambda])\right)\ ,\end{aligned} \tag{31}$$

with ${\rm Tr}\,[T^AT^B] = \delta^{AB}/2$. The boundary Lagrangian has the standard form of a four-dimensional model built from the chiral multiplet charged under the $U(1)$ symmetry, but with a crucial difference: the gauge fields (A_μ, λ_L, D) are replaced by the boundary values of the bulk fields $(A_\mu, \lambda^1_L, X^3 - \sqrt{-g^{55}}\partial_5\Phi)$.

3.1 Inflation and conventional Hubble law

In analogy with Eqs. (11) and (15), we suppose that the boundary chiral multiplets contain the fields S and $\phi_\pm$ with the corresponding Fayet-Ilopoulos D-term on our brane contained in $\mathcal{L}_4$. The Fayet-Ilopoulos D-term is now written as

$$\mathcal{L}_{\rm FY} = \left(|\phi_+|^2 - |\phi_-|^2 + \xi\right)\left(X^3 - \sqrt{-g^{55}}\partial_5\Phi\right)\ . \tag{32}$$

Again, for very large values of inflaton S, the vacuum expectation values of the fields $\phi_\pm$ are driven to zero and $\mathcal{L}_{\rm FY}$ reduces to

$$\mathcal{L}_{\rm FY} = \xi\,\left(X^3 - \sqrt{-g^{55}}\partial_5\Phi\right)\ . \tag{33}$$

This D-term will be responsible for the inflationary stage.

With the action (30), the boundary Fayet-Iliopoulos term (32) couples to the auxiliary field X^3 through the terms

$$\frac{1}{2}\int d^4x \int_{-\pi R}^{+\pi R} dy\, \sqrt{g_5}\left\{\frac{1}{g^2}{\rm Tr}\,(X^3)^2 + \delta(y)\frac{\xi}{\sqrt{-g_{55}}}\,\left(X^3 - \sqrt{g^{55}}\partial_5\Phi\right)\right\}\ . \tag{34}$$

Integrating out the auxiliary field X^3 through its equation of motion

$$X^3 + g^2\,\frac{\xi}{\sqrt{-g_{55}}}\,\delta(y) = 0 \tag{35}$$

and including the kinetic term of the field Φ, the singular terms can be rearranged into a perfect square

$$\frac{1}{2\,g^2}\int d^4x \int_{-\pi R}^{+\pi R} dy\,\sqrt{g_5}\left[\frac{1}{2}\,g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi+\frac{1}{2\,g_{55}}\left(\partial_5\Phi+g^2\,\xi\delta(y)\right)^2\right]. \tag{36}$$

At this point, it is worth emphasizing that the singular terms proportional to $\delta(y)$ and $\delta^2(y)$ play a crucial role at the quantum level since they provide counterterms which are necessary in explicit computations to preserve supersymmetry [9]. In particular, the role of the interaction term proportional to $\delta^2(y)$ is to cancel the singular behaviour induced in diagrams where the Φ-field is exchanged.

From Eq. (36) we can easily compute the energy momentum tensor of the system

$$\begin{aligned} g^2\,T_{\mu\nu}\Big|_\Phi &= \partial_\mu\Phi\partial_\nu\Phi-g_{\mu\nu}\left[\frac{1}{2}g^{\rho\sigma}\partial_\rho\Phi\partial_\sigma\Phi+\frac{1}{2}g^{55}\left(\partial_5\Phi+g^2\,\xi\delta(y)\right)^2\right], \\ g^2\,T_{55}\Big|_\Phi &= \frac{1}{2}\left(\partial_5\Phi+g^2\,\xi\delta(y)\right)^2-\frac{1}{2}\,g_{55}\,g^{\rho\sigma}\partial_\rho\Phi\partial_\sigma\Phi\,. \end{aligned} \tag{37}$$

Notice the appearance of a potentially dangerous singular terms $\delta(y)$ and $\delta^2(y)$.

The following step amounts to integrating out the heavy field Φ. The reader should remember that, since this field is odd under the discrete Z_2-symmetry, it does not have a zero mode and all its modes are as massive as the inverse of the radius of compactification. This procedure makes therefore sense in the Kaluza-Klein approach whose purpose is to give a four-dimensional interpretation of the five-dimensional world and is supposed to work when the energy scale of the system, in our case the Hubble rate, is much smaller than the inverse of the radius of the fifth dimension.

Varying this action with respect to Φ, we find that Φ satisfies the equation

$$\partial_\mu\left(\sqrt{g_5}\,g^{\mu\nu}\,\partial_\nu\Phi\right)+\partial_5\left[\frac{\sqrt{g_5}}{g_{55}}\left(\partial_5\Phi+g^2\,\xi\delta(y)\right)\right]=0\,. \tag{38}$$

We now look for solutions such that a and n are independent of the fifth coordinate y, such as $a(t,y)=a(t)$. We can also fix $n(t)=1$ and suppose that the radion is fixed at the minimum of its potential.

Since Φ is an odd field under the Z_2-parity we have $\Phi(0)=\Phi(\pi R)=0$ (where, for instance, $\Phi(0)$ has to be intended as $(\Phi(0^+)+\Phi(0^-))/2$). For a static solution $\partial_\mu\Phi=0$, these boundary conditions of the field Φ require that $\partial_5\Phi$ must integrate to zero around the circle

$$\partial_5\Phi=-g^2\xi\left(\delta(y)-\frac{1}{2\pi R}\right)\,. \tag{39}$$

Substituting this solution into the Lagrangian (36) one finds that the various singular terms cancel and one is left with the usual D-term interaction

$$S=-\frac{1}{2}\int d^4x\int_{-\pi R}^{+\pi R} dy\,\sqrt{g_4}\,\frac{1}{b_0}\,\frac{g^2}{2}\left(\frac{\xi}{2\pi R}\right)^2=-\int d^4x\,\sqrt{g_4}\,\frac{\bar{g}^2\xi^2}{4} \tag{40}$$

where $\bar{g}^2 = (g^2/2\pi b_0 R)$. Correspondingly, the energy momentum (37) tensor reduces to

$$\begin{aligned} T_{\mu\nu}|_\Phi &= \frac{g_{\mu\nu}}{\pi b_0 R} V \,, \\ V &= \frac{1}{4}\bar{g}^2\xi^2 \,. \end{aligned} \tag{41}$$

Making use of the Lagrangian (4) with

$$R_4 = -6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) \tag{42}$$

or, equivalently, using the (00)-component of Einstein's equations, we find that the conventional four-dimensional Hubble law governing the expansion rate of the Universe

$$H^2 = \frac{1}{3}\frac{V}{M_4^2} \,, \tag{43}$$

where $M_4^2 = \pi b_0 R M_5^3$ is the reduced four-dimensional Planck mass. One can also easily show that the (55)-component of Einstein's equations is satisfied once the shift in the radion vacuum expectation value is taken into account.

This result is quite gratifying. The singular terms proportional to $\delta(y)$ and $\delta^2(y)$ disappear after we substitute in the 5D Lagrangian the solution of the classical equation of motion for the heavy field Φ. The remarkable consequence is that the energy-momentum tensor is not peaked around the brane at $y = 0$ and the constraint (18) needs not to be imposed. Conventional 4D evolution is recovered. The source of such findings is manifest: supersymmetry imposes the presence of the bulk propagating field Φ in the effective auxiliary D-term on the boundary. At the level of the effective 4D theory, all singular terms disappear after we substitute in the Lagrangian the solution of the classical equation of motion of such odd field Φ. Dynamically what happens is that the bulk field Φ adjusts itself to response to any change in the D-term on the boundary. This phenomenon is responsible for the redistribution of the energy density in the bulk and for removing the singular terms at the boundary.

We close this section by reminding the reader that our finding hold as well in the case in which the inflationary stage is driven by an F-term. In such a case, instead of a vector multiplet in the bulk, one starts from a supersymmetric 5D hypermultiplet in the bulk. The latter contains two scalar fields A_1 and A_2 which are even and odd respectively under the Z_2 symmetry. The F-term on the boundary is

$$F_1 - \sqrt{-g^{55}}\partial_5 A_2 \tag{44}$$

leading to a boundary action

$$\mathcal{L}_4 = \left(F_1 - \sqrt{-g^{55}}\partial_5 A_2\right)\frac{\partial W}{\partial A_1} \,. \tag{45}$$

where W is the boundary superpotential and $V_F = \left|\frac{\partial W}{\partial A_1}\right|^2$ is the F-term vacuum energy density resposible for inflation. Integrating out the auxiliary field F_1 one finds the bulk action

$$S = \frac{1}{2}\int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{g_5} \left[g^{\mu\nu}\partial_\mu A_2^* \partial_\nu A_2 + \frac{1}{g_{55}}\left(\partial_5 A_2 - \left|\frac{\partial W}{\partial A_1}\right| \delta(y)\right)^2\right]. \qquad (46)$$

Again, one can solve the equation of motion for the odd bulk field A_2 and show that the singular terms proportional to $\delta(y)$ and $\delta^2(y)$ are cancelled leaving behind an energy momentum tensor $T_{\mu\nu}|_{A_2} = \frac{g_{\mu\nu}}{2\pi b_0 R} V_F$. The latter guarantees the standard four-dimensional Hubble expansion during inflation.

4 Conclusions and directions for future work

We have shown that, if we start with a vacuum energy density confined on our brane and the bulk is supersymmetric, the back-reaction of the bulk supersymmetric fields smooth out the singularities which would be otherwise displayed in the energy-momentum tensor of the system. This considerably simplifies the solution of Einstein's equations since derivatives of the metric do not jump across the brane, ensuring that uniform solutions along the fifth dimension can be found. Conventional 4D evolution during the inflationary stage is recovered without resorting to any nontrivial configuration of the scale factors along the extra-dimension and to any limitation in the bulk, $|y| \leq |y_m|$.

Our results suggest that the study of cosmological phenomena in the brane world scenario must be performed including all ingredients provided by supersymmetry. This comment is particularly relevant when applied to gravity. In supersymmetric brane world scenarios, even though chiral matter and gauge fields may be restricted to live on the boundaries, gravity always propagates in the bulk. Using an off-shell supergravity multiplet one can integrate out the auxiliary fields and examine the couplings between the on shell bulk supergravity fields and boundary matter fields [10, 11, 12]. The (super)gravity on shell multiplet contains apart from the fünfbein e_M^A and the symplectic Majorana gravitino Ψ_M, the graviphoton A_M. The situation is analogous to what happens in the case of an off-shell bulk vector multiplet in 5D analyzed in section 3. There, the presence of the propagating odd field Φ in the effective D-term $D = (X^3 - \sqrt{-g^{55}}\partial_5\Phi)$ on the boundary induced new interactions between the the chiral matter fields living at the boundary and the Φ field. In supergravity one finds new interaction terms at the boundaries (compared to the usual ones in $N = 1$ 4D supergravity coupled to chiral or vector multiplets) involving the components of the chiral and vector multiplets and the graviphoton field strength $F_{\mu 5}$. The latter is made out of the five-dimensional field A_μ (odd under the Z_2 symmetry) and the four-dimensional field A_5 which plays the role of the imaginary part of the radion modulus. If a chiral multiplet $(\varphi, \psi_\varphi, F_\varphi)$ lives on the boundary, the graviphoton couples to the current $J^\mu = i(\varphi^\dagger \partial^\mu \varphi - \varphi \partial^\mu \varphi^\dagger) + \psi_\varphi \sigma^\mu \bar{\psi}_\varphi$ forming again a perfect square [12]

$$S = \frac{1}{4} \int d^4x \int_{-\pi R}^{+\pi R} dy \, \sqrt{g_5} \left(\sqrt{-g^{55}} \, F_{\mu 5} - i \sqrt{\frac{3}{2}} \, J_\mu \, \delta(y) \right)^2 . \tag{47}$$

If during the evolution of the Universe the current acquires a vacuum expectation value, one may not disregard the axion-like coupling of the graviphoton field with the current. On the contrary the latter acts as a source for the graviphoton. This happens, for instance, in the Affleck-Dine scenario [13] where the generation of the baryon asymmetry is induced by time-dependent baryonic currents of scalar fields, or in the presence of topological defects around which the imaginary part of the scalar field the defect is made of winds.

The same arguments tell us that the phenomenon of reheating after inflation cannot be a purely four-dimensional event since the release of the vacuum energy density is accompanied by large fluctuations of the bulk fields. For instance, in the D-term inflationary scenario, the bulk quantity $\partial_5 \Phi$ changes from $V^{1/2}$ during inflation to zero after inflation. The same large fluctuations may occur during a primordial phase transition. All these issues deserve further study and are currently under investigation.

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